

# An application of electrothermal feedback for high resolution cryogenic particle detection

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A novel type of superconducting transition edge sensor is proposed. In this sensor, the temperature of a superconducting film is held constant by feeding back to its position on the resistive transition edge. Energy deposited in the film is measured by a reduction in the feedback Joule heating. This mode of operation should lead to substantial improvements in resolution, linearity, dynamic range, and count rate. Fundamental resolution limits are below  $\Delta E = \sqrt{kT^2 C}$ , which is sometimes incorrectly referred to as the thermodynamic limit. This performance is better than any existing technology operating at the same temperature, count rate, and absorber heat capacity. Applications include high resolution x-ray spectrometry, dark matter searches, and neutrino detection. © 1995 American Institute of Physics.

Superconducting transition edge sensors suffer from limitations due to film nonuniformity, transition nonlinearity, and limited dynamic range. In electrical circuits, analogous limitations are often avoided by using feedback techniques. This scheme can be used with superconducting transition edge sensors by using feedback to hold the temperature of the film constant. The feedback signal is the required heat input.

Joule heating can be used to provide such feedback. When a superconducting film is voltage biased and the substrate is cooled to well below the transition temperature, the film can be made to self-regulate in temperature within its transition. As the film cools, its resistance drops towards zero, and the Joule heating increases. A stable equilibrium is established when Joule heating matches the heat loss into the substrate. The feedback signal is the change in Joule power in the film, the product of the bias voltage, and the measured current.

It has been noted that when films are voltage biased, pulse duration can be shortened.<sup>1-3</sup> If film transitions are sharp and the substrate is cooled to well below the transition of the film, this effect should be quite large.

We model the detector as a film of heat capacity  $C$  connected to a substrate at a constant temperature  $T_s$ . The temperature of the film is determined by the Joule heating and the heat loss to the substrate. The heat loss to the substrate goes as  $K(T^n - T_s^n)$ , where  $K$  is a material and geometry dependent parameter and  $n$  is a number whose value depends on the dominant thermal impedance between the substrate and the electrons in the superconducting film. This thermal impedance is set at higher temperatures and in thicker films by the Kapitza boundary resistance between the film and the substrate. For thinner films, and at lower temperatures, the electron-phonon decoupling in the film dominates. If the Kapitza resistance dominates,  $n$  is 4. If electron-phonon decoupling in the film dominates,  $n$  is either 5 or 6, depending on the theory and the temperature range.

When an event heats the film a small amount above the

equilibrium temperature, the return to equilibrium is described by

$$C \frac{d\Delta T}{dt} = -\frac{P_0 \alpha}{T} \Delta T - g \Delta T, \quad (1)$$

where the first term on the right-hand side is the effect of reduced Joule heating, and the last term is the effect of increased heat flow to the substrate. The thermal conductance  $g = dP/dT = nKT^{n-1}$ ,  $\alpha = (T/R)(dR/dT)$ , a unitless measure of the sharpness of the superconducting transition, and  $P_0$  is the equilibrium Joule power. When the substrate temperature is much colder than the film,  $P_0 = KT^n = gT/n$ , and the pulse recovery time constant is seen to be

$$\tau_{\text{eff}} = \frac{\tau_0}{1 + \alpha/n}, \quad (2)$$

where  $\tau_0 = C/g$  is the intrinsic time constant of the film (the pulse recovery time constant in the absence of Joule heating).

Extremely low- $T_c$  superconducting films can be fabricated with  $\alpha$  as high as 1000.<sup>4</sup> Thus, electrothermal feedback can shorten pulse duration by two orders of magnitude. Under these conditions, major simplifications are possible in interpreting detector pulses.

When the pulse duration is much shorter than the intrinsic time constant, the detector is operating in a mode where the pulse energy is removed by a reduction in Joule heating, instead of an increased heat flow to the substrate. Thus, the energy deposited in the film is simply the integral of the change in the Joule power

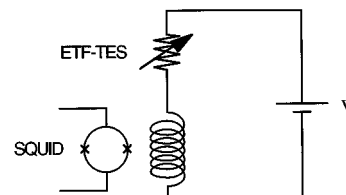


FIG. 1. Instrumentation of the ETF-TES.

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$$E = \int \Delta P_{\text{Joule}} dt = V_0 \int \Delta I dt. \quad (3)$$

This observation leads to a simplification in pulse analysis. Nonuniformities in the film properties and nonlinearities in the resistive transition will effect the pulse shape, but not the pulse integral. In addition, the dynamic range is improved. If an energetic event drives the film completely normal, the total energy is still measured. The saturation causes the pulses to lengthen, but energy loss is small as long as the pulse duration is still much smaller than  $\tau_0$ .

This technique allows a direct measurement of energy collection efficiency. The energy is simply the voltage bias multiplied by the integral of the current change. Since there is no proportionality constant, this measurement has no free parameters, and can be compared directly to the incident energy.

In its simplest form, the electrothermal feedback transition edge sensor (ETF-TES) is a superconducting film on a silicon substrate. The film is voltage biased, and the current is measured with a SQUID (Fig. 1).

The inductance of the SQUID input coil sets a practical lower limit on the pulse duration. The  $L/R$  time constant (from the SQUID input coil inductance and the resistance of the film) must be kept short compared to the effective pulse duration. If this is not done, resonant oscillations can occur which involve the electrothermal feedback and the SQUID inductance. This effect limits the minimum pulse duration to  $\sim 1 \mu\text{s}$ . Thus, to operate deep in the feedback regime, intrinsic time constants must be at least  $100 \mu\text{s}$ . The bandwidth of the SQUID need not further limit the pulse duration, as 175 MHz bandwidths have been achieved with dc SQUID arrays.<sup>5</sup>

In order to achieve intrinsic time constants on the  $100 \mu\text{s}$  time scale, the film transition must be below about 100 mK. Superconducting tungsten films have been deposited with transitions near 70 mK, and with  $\alpha$  as high as 1000.<sup>4</sup> Intrinsic time constants for these films were experimentally found to be above  $100 \mu\text{s}$ .

The energy resolution of an ETF-TES is ultimately limited by thermodynamic energy fluctuations in the detector (phonon noise), and the Johnson noise of the film. These limitations have been analyzed for other devices.<sup>2</sup> Here we extend this analysis to the extreme feedback regime of our transition-edge sensors, leading to an important improvement.

The detector is modeled as a heat capacity  $C$  attached to a substrate. The power flow equation is

$$C \frac{dT}{dt} = \frac{V^2}{R(T)} - K(T^n - T_s^n) + P(t), \quad (4)$$

where  $P(t)$  is the power flow from the substrate due to phonon noise. If we consider small signals and look at one Fourier component, we find that  $T_\omega = (I_0 V_n + P_n) / (P_0 \alpha / T_0 + i\omega C)$ . The term proportional to the Johnson noise voltage  $V_n$ , describes the interaction of the Johnson noise with the electrothermal feedback.  $P_n$  is the phonon noise associated with  $g$ . We have used  $gT \ll \alpha P_0$ , which is valid in our feedback limit. The power spectrum of the Johnson noise is

$V_n^2 = 4kTR_0$ . The phonon noise is  $P_n^2 = 2kT^2g$ , since  $T_s^n \ll T^n$ . The SQUID signal is  $\Delta I = \Delta(V/R) = \Delta V/R - I_0 \alpha \Delta T/T_0$ . After some computation, we arrive at the noise current spectral density

$$I_n^2 = \frac{4kT}{R_0} \frac{n^2/\alpha^2 + \omega^2 \tau_{\text{eff}}^2}{1 + \omega^2 \tau_{\text{eff}}^2} + \frac{4kT}{R_0} \frac{n/2}{1 + \omega^2 \tau_{\text{eff}}^2}. \quad (5)$$

Here, the first term is the Johnson noise. It is seen that the electrothermal feedback suppresses the Johnson noise for frequencies short compared to  $1/\tau_{\text{eff}}$ , an effect first noted by Mather.<sup>1</sup> The second term is the phonon noise, which rolls off at high frequencies. It should be noted that the temperature in this equation is the temperature of the film, not the temperature of the substrate. The substrate need not be very much colder than the film, however, before most of the pulse shortening occurs.

When the optimal filter is applied to an exponential pulse in the presence of this noise spectrum, the fundamental resolution limit is calculated to be

$$\Delta E_{\text{FWHM}} = 2.36 \Delta E_{\text{rms}} = 2.36 \sqrt{4kT^2 C (1/\alpha) \sqrt{n/2}}. \quad (6)$$

Thus, ETF-TES detectors might ultimately have resolutions below what is erroneously called the thermodynamic limit ( $\Delta E = \sqrt{kT^2 C}$ ) by a factor of  $\sim 2/\sqrt{\alpha}$ . This resolution is of the same order as the ultimate resolution limit obtainable in principle with superconducting transition edge sensors of the same  $\alpha$  operated in a conventional (nonfeedback) mode, but there is a significant difference in rate.

It has been observed that conventional superconducting transition edge sensors encounter an important limit as  $\alpha$  is increased.<sup>6</sup> To get resolutions that improve as  $1/\sqrt{\alpha}$ , the signal bandwidth must be increased linearly with  $\alpha$ . The optimal filter must be able to use frequencies well past the  $\omega = 1/\tau_0$  knee in the signal. When the bandwidth encounters the time required to thermalize the energy of an event, however, resolution can be improved no further unless the detector is made slower.

When a sensor is run in the extreme feedback mode, this limit is not encountered. Most of the sensitivity is at frequencies below the  $\omega = 1/\tau_{\text{eff}}$  knee. Resolutions still improve as  $1/\sqrt{\alpha}$ , even if the detector speed is near the thermalization time. Therefore, ETF-TES devices may be made much faster than conventional devices of similar resolutions, with the same electronics bandwidth.

Note, that if film nonuniformities, transition nonlinearities, or saturation of the film due to large current pulses cause significant differences in the pulse shape, it is difficult to apply the optimal filter. Large current pulses might also affect the order parameter, changing the transition shape. The optimal-filter resolution calculated above is not strictly applicable in these cases. Fortunately, unlike conventional transition edge sensors, most of the information in an ETF-TES is contained in frequencies below  $\omega = 1/\tau_{\text{eff}}$ , so good results should be obtainable with nonoptimal filtering techniques.

The high resolution and high count rate of ETF-TES sensors suggest their use as high resolution x-ray detectors. In this application, it is desirable to make the film heat capacity large enough so that the highest expected incident

energy drives the film through about one tenth of a transition width. As long as  $\tau_{\text{eff}} \ll \tau_0$ , energy depositions exceeding this condition will still be correctly measured, but the pulse duration will begin to lengthen. For a tungsten film with a 70 mK transition, and heat capacity chosen so that a 6 keV x-ray drives it through one tenth of a transition width, the fundamental limit on energy resolution is less than 1.2 eV FWHM. If the film has  $\alpha=1000$  when biased at one tenth of the normal resistance, this heat capacity corresponds to an area of about 1 mm $\times$ 1 mm, and a thickness of 1  $\mu$ m, enough to absorb  $\sim$ 50% of incident 6 keV x rays.

If a larger absorber volume is desired, a low heat capacity semimetal such as bismuth can be thermally connected to the superconducting film, and used as an absorber.<sup>7</sup> Bismuth dimensions of 2  $\mu$ m $\times$ 1.6 cm $\times$ 1.6 cm would also absorb  $\sim$ 50% of incident 6 keV x rays, while limiting temperature excursions to one tenth of the transition width.

In the application of x-ray spectrometry to materials analysis, Poisson statistics determine the uncertainty in x-ray emission line peak heights. The minimum detectability limit goes as the square root of the ratio of the energy resolution to the count rate.<sup>8</sup> Energy dispersive spectrometry (EDS) is typically limited to a 150 eV resolution and a 20 kHz count rate for 6 keV x rays. Cryogenic calorimeters are theoretically limited to a 1 eV resolution and a 1 kHz count rate, yielding uncertainties 3 times better than EDS.<sup>8</sup> ETF-TES sensors are theoretically limited to about 1.2 eV resolution with count rates above 50 kHz, leading to a potential performance  $\sim$ 15 times better than EDS.

The high potential resolution of ETF-TES detectors makes their use interesting in phonon-mediated cryogenic particle detectors for dark matter searches and neutrino scattering experiments. If many widely spaced film segments are voltage biased in parallel, phonon energy can be collected

over a very large area via quasiparticle trapping from aluminum pads.<sup>9</sup> Variations in film properties are unimportant as each segment would self-bias near the center of its transition. An ETF-TES phonon-mediated particle detector effort is currently under way.<sup>10</sup>

In summary, the ETF-TES provides important advantages. By operating in a feedback mode, it reduces nonlinearities and nonuniformities, and has fundamental resolution limitations lower than any existing technology operating at the same temperature, count rate, and absorber heat capacity. Practical limitations, including flux flow noise on the transition edge, and statistical variations in incident energy thermalization, have yet to be determined.

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